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Obtaining the radiation resistance of a piston by measuring sound pressures only:

The Moving Microphone System (MMS).

by

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Abstract

The Moving Microphone System (MMS) is a method to measure the radiation resistance seen by a loudspeaker diaphragm. The basic measurement of radiation resistance calls for two absolute measurements: the velocity of the diaphragm and the force acting on the surface of the diaphragm. The drawback of such systems is that the ratio of the transfer functions of the two transducers involved should be kept very constant throughout the lifetime of the transducers unless frequent calibrations are involved. The proposed system reduces the required period of constant transfer function(s) to the order of one minute or less. In one embodiment of the invention only one transducer (i.e. microphone) is used to measure the sound pressure in two positions near the surface of the diaphragm: first the microphone is measuring at position #1 then the microphone is physically moved to position #2 to measure the sound pressure here. Alternatively two microphones could be employed, and in this case the two microphones physically swap position, and then the measurements are repeated. When using two microphones one might also prefer a separate calibration combined with one measurement of the two sound pressures, which only requires one microphone to physically move. Measurement of the sound pressures at three or more positions is considered as an extension of measuring two sound pressures. The advantages of all embodiments is that the need of highly stable absolute measurements is changed to a need for relative measurements.

Contents:

1 Radiation resistance

When a diaphragm/piston is moving at a velocity, v, then a pressure, p, arises in front of the diaphragm. Considering the pressure on the surface of the diaphragm leads to a force acting on the diaphragm. The ratio between a force and a velocity is referred to as a mechanical impedance. Since this force arises due to a sound pressure, which in turn arises due to the radiated sound field acting back on to the diaphragm, this particular impedance is referred to as a radiation impedance [1].

The radiation impedance is the mechanical load seen from the diaphragm into the surroundings. A radiation impedance is a complex number dependent on frequency, i.e. comprising both a real and an imaginary number at each frequency. The imaginary part corresponds to a portion of air simply moving along with the diaphragm like an additional mass added to the mass of the diaphragm. The real part on the other hand corresponds to the loss of energy consumed by radiating acoustic power into the surroundings. This real part of the radiation impedance is termed the radiation resistance, R_r.

Depending on which units are preferred, the radiation resistance can be regarded as a mechanical resistance or an acoustic resistance. The acoustic radiation impedance is calculated as the ratio between sound pressure, p, and volume velocity, q. The relationship between acoustic radiation resistance and mechanical radiation resistance is usually simple: a factor equal to the squared area of the diaphragm/piston. In this context they are considered equal and simply referred to as radiation resistance (R_r).

2 Measuring radiation resistance

Radiation resistance can be measured in a number of ways. Usually radiation resistance demands measurement of the motion of the loudspeaker diaphragm (displacement, velocity or acceleration etc.) and force (from the sound field) acting on the diaphragm. If the diaphragm can be considered as a piston, then the velocity is equal everywhere on the diaphragm and the velocity can be measured at one point only. Measuring the force acting on the surface of the diaphragm/piston requires a measurement of the sound pressure at "every point" on the surface and integrating these sound pressures to obtain the force. Then the mechanical radiation resistance, $R_r(f)$, can be calculated from equation 1, by dividing the force, $F(f)$, by the velocity, $v(f)$, and isolating the real part of this ratio (Re(x) = the real part of x). Please note the dependence of the frequency, f, for R_r, F and v.

$$R_r(f) = Re\left(\frac{F(f)}{v(f)}\right) \tag{1}$$

Considering an infinitely small part of the piston, of area dA, mounted in an infinite baffle (2π conditions), then the sound pressure, dp, at a distance, r, can be calculated from equation 2. Where c is the speed of sound, ρ is the density of air and j is the square root of -1. Further detail can be found in [1].

$$dp(f) = j \cdot f \cdot \rho \cdot \frac{v(f)}{r} \cdot dA \cdot e^{-j\,2\pi f \frac{r}{c}} \tag{2}$$

Considering equation 1, leads to a change of equation 2 into equation 3, where the velocity, $v(f)$, is divided out and the real part is isolated. This operation requires that the diaphragm is acting like a piston i.e. equal velocity at all points on the surface of the diaphragm.

$$Re\left(\frac{dp(f)}{v(f)}\right) = 2\pi \cdot f^2 \cdot \frac{\rho}{c} \cdot dA \cdot \frac{\sin\left(2\pi f \frac{r}{c}\right)}{2\pi f \frac{r}{c}} \tag{3}$$

A very important characteristic of equation (3) is that the last factor is a sin(x)/x type of factor, which reduces to unity if x is sufficiently small. For example, if 1 dB is set as the maximum error allowed, then the distance, r, must obey equation 4, where λ is the wavelength. If equation 4 is obeyed, then equation 3 reduces to equation 5.

$$r < \frac{\lambda}{8} \tag{4}$$

$$Re\left(\frac{dp(f)}{v(f)}\right) = 2\pi \cdot f^2 \cdot \frac{\rho}{c} \cdot dA \tag{5}$$

The important characteristic of equation 5 is, that provided that equation 4 is obeyed, then the real part of the sound pressure divided by the velocity is independent of distance! I.e. if the distance, *r*, is less than one-eighths of the wave length, then the real part of the sound pressure divided by the velocity of the piston is independent of the actual distance - even at zero distance. Here it should be stressed, that dividing by the velocity of the piston makes the velocity of the piston the phase reference i.e. the phase of the velocity of the piston is defined as zero phase.

If the piston under consideration is small compared to the wave length, according to equation 4, then the real part of the sound pressure at any point on the surface of the piston can be calculated using equation 5. The total force acting on an infinitely small part of the area, dA_p, is calculated by integrating equation 5 across the total area of the piston, i.e. the particular area, dA_p, is under influence from any other infinitely small part of the piston - even from dA_p itself.

But since the contributions are independant of distance, then all the infinitely small areas of the piston contributes equally to the total sound pressure, p, at the particular area, dA_p. This is the reason, why the integration of equation 5 is trivial: *dA* integrated over the area of the piston simply yields the area of piston: *Area*. A total sound pressure on the surface of the considered area, dA_p, corresponds to an infinitely small force, *dF*, acting on this particular area, dA_p. Equation 6 states how the real part of the force on an infinitely small area is calculated, provided that equation 4 is fulfilled for all distances on the piston. The total area of the piston is *Area*. Like the pressure was relative to the velocity, so is the infinitely small force, dF(f).

$$Re\left(\frac{dF(f)}{v(f)}\right) = Re\left(\frac{dp(f)}{v(f)}\right) \cdot dA_p = \left(2\pi \cdot f^2 \cdot \frac{\rho}{c} \cdot Area\right) \cdot dA_p \tag{6}$$

The total force acting back from the sound field onto the surface of the piston can now be found by integrating equation 6 across the total area of the piston. Again this integration becomes trivial, provided that equation 4 is obeyed. The infinitely small force is equal at all points, which leads to the strait-forward result that dA_p integrates to the total area of the piston, *Area*. These are the prerequisites of equation 7, which states how the real part of the total force divided by the velocity for a piston under 2π conditions is calculated.

$$Re\left(\frac{F(f)}{v(f)}\right) = 2\pi \cdot f^2 \cdot \frac{\rho}{c} \cdot Area^2 \tag{7}$$

Substituting equation 7 into equation 1 leads to equation 8, which gives the radiation resistance for a piston under 2π conditions, provided that equation 4 is obeyed.

$$Rr(f) = 2\pi \cdot f^2 \cdot \frac{\rho}{c} \cdot Area^2 \tag{8}$$

When the radiation resistance is to be measured, then equation 6 plays a central role compared to equation 8. The real part of the total sound pressure relative to the velocity at any point on the surface of the piston or in front of the piston is equal as long as equation 4 is obeyed. This sound pressure only lags a factor equal to the piston area, *Area*, to be turned into equation 8. And according to equation 6, this sound pressure can be measured using a microphone positioned at any point in front of the piston as long as equation 4 is obeyed.

Figure 1 depicts one suggestion for a measurements setup. In this example an accelerometer is used to detect the motion of the diaphragm/piston. To calculated the radiation resistance, the piston velocity is needed, which can be calculated from the acceleration by integrating once.

Figure 2 shows the principle elements in the measurement chain relating to figure 1. Unit A is a power amplifier. Unit B is the electrical and mechanical parts of the loudspeaker driver unit and the acoustic influence of the cabinet enclosure. Unit C is the transfer function from membrane acceleration to sound pressure in front of the membrane. Unit D is the microphone and unit E is the accelerometer. Signal 1 is the input voltage at line level, and signal 2 is the voltage on the terminals of the loudspeaker driver unit. Signal 3 is the acceleration of the membrane, and signal 4 is the sound pressure at some point in front of the driver. Signal 5 is a voltage corresponding to the measured sound pressure, and signal 6 is a voltage corresponding to the measured acceleration.

The radiation resistance, $R_r(f)$, can be calculated from equation 9, in which it should be noted that the real part of unit C differentiated is the radiation resistance, except for the factor, Area, which is the area of the diaphragm, according to the definition of mechanical radiation resistance. Note that R_r is a function of frequency, *f*, and the multiplication by s (Laplace operator) originate in the integration of the acceleration mentioned above. Acoustic radiation resistance is equivalent to mechanical radiation resistance in this context.

$$R_r(f) = Re\left(\frac{p(f)}{v(f)} \cdot Area\right) = Re(\, s \cdot C(f) \cdot Area\,) \qquad (9)$$

Equation 10 states how the radiation resistance can be approximated by *y(f)* from measurements of the transfer functions in figure 2. What is actually measured is the sound pressure, *signal 5*, and the acceleration of the diaphragm/piston, *signal 6*. The approximation in equation 10 compared to equation 9, is the ratio of D and E, i.e. the ratio of the transfer functions of the microphone and the accelerometer.

$$\begin{aligned} y(f) &= Re\left(s \cdot \frac{signal\,5}{signal\,6}\right) = Re\left(s \cdot \frac{A(f) \cdot B(f) \cdot C(f) \cdot D(f)}{A(f) \cdot B(f) \cdot E(f)} \cdot Area\right) \\ &= Re\left(s \cdot C(f) \cdot Area \cdot \frac{D(f)}{E(f)}\right) \end{aligned} \qquad (10)$$

Since a microphone and an accelerometer can not be considered as ideal transducers, a calibration is normally needed. A relative calibration of the two transducers is sufficient, since it is the ratio of D and E that influences the measurement.

It is a known fact that the sound pressure near a sound source at low frequencies is almost 90 degrees out of phase with the velocity of the diaphragm/piston. This can be illustrated in the complex plane, where the velocity is taken as the phase reference, i.e. the velocity coincides with the real axis (Re), see figure 3. The real part of the sound pressure relative to the velocity is shown in the same figure: Re(p). Re(p) is in fact the same as the radiation resistance, R_r, if Re(p) is divided by the modulus of the velocity, v.

From this it becomes evident why the measurements of the phases of both sound pressure and velocity of the diaphragm/piston must be performed with a very high precision, because even small changes of the phase of sound pressure, p, relative to the velocity, v, can effect Re(p) and in turn the radiation resistance, R_r.

This demand for a very high phase accuracy applies for the relative calibration of the two transducers (e.g. microphone and accelerometer), which must stay valid for the entire life cycle for a particular measurement setup. For example, if a measurement setup should measure the radiation resistance of a diaphragm/piston under a number of different conditions, such as free field radiation, large rooms, small rooms etc., then the transducers must keep a constant relative calibration throughout all the different measurements. Should this not be the case, then frequent calibrations of the transducers must be performed. Causes for a non stable relationship between the two transducers could be their different reactions to temperature changes, changes of humidity, changing static air pressure etc.

When the radiation resistance is to be measured under other conditions than 2π conditions, e.g. a typical rectangular listening room with 6 reflective surfaces, then the real part of the sound pressure arising on the surface of the piston divided by the velocity of the piston due to individual reflections should be considered.

Figure 4 shows how one individual reflection contributes to the total radiation resistance depending on the parameter, *x*, which is basically the product of frequency, *f*, and the distance, *d*, that the reflection has travelled, i.e. from the piston to the wall(s) and back to the surface of the piston. Figure 4 is based on equation 3, which states how the real part of a sound pressure relative to the piston velocity is calculated depending on the distance, *r*. Equation 11 defines *x* from the frequency, *f*, distance, *d*, and the speed of sound, *c*.

$$x = \frac{2\pi}{c} \cdot f \cdot d \tag{11}$$

As argued above, one microphone can be used to measure the sound pressure somewhere near the surface of the piston. When considering the contribution from the reflections, then the distance, *d*, is reduced by a distance, *a*, which is the distance between microphone and the surface of the piston, i.e. the reflection has not reached the surface of the diaphragm/piston when it reaches the microphone. The effective *x* therefore becomes x_e as defined in equation 12.

$$x_e = \frac{2\pi}{c} \cdot f \cdot (d - a) \tag{12}$$

If the distance from the piston surface to the microphone, *a*, is small compared to the distance traveled by the reflection, *d*, then x_e and *x* will be very similar at any value of the frequency, *f*. It follows from this, that reflections, which has traveled a distance large compared to the distance from the microphone to the surface of the piston, are measured correctly.

If a can't be considered small compared to *d*, then figure 4 should be examined in detail: for values of *x*, where the derivative of the curve in figure 4 is small (at non-steep parts of the curve), then a change of *x* to x_e will not effect the contribution to the total radiation resistance in a significant way, e.g. below x=1, between 4 and 5, between 7 and 8 etc.

But for values of *x*, which corresponds to the steep parts of the curve in figure 4, care should be taken in choosing a distance to the microphone, which is sufficiently small to meet the accuracy needed in the measurements. The error is maximized at approximately x=3 and at the maximum frequency of interest in the measurements, f_{max}. Equation 13 states the approximately maximized error in dB as a function of the maximum frequency, f_{max}, the speed of sound, *c*, and the distance between microphone and the surface of the piston, *a*. Please note, that equation 13 is only valid if ($a \cdot f_{max}$) is less than approximately 100.

$$Error_{Max} = 20 \cdot \log\left(\frac{1 + \frac{\sin(3)}{3}}{1 + \frac{\sin\left(3 - \frac{2\pi \cdot fmax \cdot a}{c}\right)}{3 - \frac{2\pi \cdot fmax \cdot a}{c}}}\right) \tag{13}$$

Considering both the measurement of the contributions from the piston itself and from the reflections to the total radiation resistance, a central problem remains: the demand for two absolute measurements, which should maintain their relative calibration throughout the lifetime of the transducers, unless frequently calibrations are employed.

3 The principle of Moving Microphone System (MMS)

The basic idea of the MMS is to use the same transducer to measure both the movement of the diaphragm/piston and the sound pressure in front of the diaphragm/piston. This idea requires the measurement of the sound pressure at two or more positions. So the idea is not bounded to two positions, but in section 3.1 to 3.5 two positions is taken as an example. Section 3.6 will then deal with three or more positions.

3.1 Using one microphone and approximating the acceleration by a sound pressure

The first approach is to use one microphone to measure the sound pressure at two positions in front of the diaphragm/piston, i.e. first placing the microphone in position #1, measuring the sound pressure, $p_1(f)$, and then moving the microphone physically to position #2, measuring the pressure, $p_2(f)$. This procedure is illustrated in figure 5.

If both position #1 and position #2 obey equation 4, then the real parts of $p_1(f)/v$ and $p_2(f)/v$ are equal, as stated in equation 14. Provided that the positions are not symmetrical placed around the axis of the diaphragm/piston, then the amplitude/modulus of $p_1(f)/v$ and $p_2(f)/v$ are different. This is illustrated in figure 6, where the amplitudes are quite different, but the real parts are equal. It should be stressed, that in figure 6, p_1 and p_2 denote the sound pressures in position #1 and #2 divided by the velocity, v, i.e. the velocity is the phase reference (coincident with the Real axis in the complex plane).

$$Re\left(\frac{p_1(f)}{v(f)}\right) = Re\left(\frac{p_2(f)}{v(f)}\right) \tag{14}$$

If the amplitude/modulus of $p_1(f)$ is much larger than $p_2(f)$, then $p_1(f)$ approaches the imaginary axis (Im) compared to $p_2(f)$. This can be obtained by choosing position #1 much closer to the surface of the diaphragm/piston than position #2, which follows from equation 2 due to the division by the distance, r.

Provided that $p_1(f)$ approaches the imaginary axis (Im), i.e. the imaginary part of $p_1(f)$ dominates the real part, then $p_1(f)$ might be used as an approximation for the velocity if $p_1(f)$ is turned 90 degrees clockwise, i.e. divided by j (square root of -1). The placement very near to the surface of the diaphragm/piston ensures, that the contributions from reflections in a reflective environment does not have a strong influence on the amplitude/modulus of the total sound pressure due to the high amplitude/modulus of the direct sound from the surface of the diaphragm/piston.

It follows from this, that the phase of $p_1(f)$ can be used to approximate the phase of the velocity simply by turning the phase of $p_1(f)$ 90 degrees clockwise, and the amplitude/modulus of the velocity can be approximated by the amplitude/modulus of $p_1(f)$. The amplitude/modulus of $p_1(f)$ is directly the amplitude/modulus of the velocity except for frequency, f, and a constant, which relates to the physical distance from the surface of the diaphragm/piston. For this reason, position #1 should be reproduced with great detail whenever measuring, i.e. moving the microphone from position #1 to position #2 and back to position #1.

Given the needed accuracy, one can calculate how high errors one can tolerate on the measured phase of the radiation impedance, and in turn how much larger the imaginary part should be compared to the real part of the radiation impedance. Equation 15 shows how the radiation resistance can be approximated by $R_{r,a}(f)$ using $p_1(f)$ to approximate to velocity of the diaphragm/piston. The origin for equation 15 is equation 9 and figure 2. Block C and D in figure 2 splits into: C_1 and D_1 when measuring $p_1(f)$, and C_2 and D_2 when measuring $p_2(f)$. Note, that $p_1(f)$ is integrated (divided by $s = j2\pi f$, i.e. the Laplace operator), which follows from the

fact that sound pressure is proportional to the acceleration of a diaphragm/piston and not the velocity. This is also the reason why $p_1(f)$ has to be turned 90 degrees to approximate the phase of the velocity (and this is done by dividing $p_1(f)$ by the pure imaginary number s as seen in equation 15). $p_{1,m}(f)$ and $p_{2,m}(f)$ denotes the measured sound pressures at position #1 and position #2.

$$
\begin{aligned}
Rr,a(f) &= Re\left(\frac{p_{2,m}(f)}{\frac{1}{s}\cdot p_{1,m}(f)}\cdot Area\right)\\
&= Re\left(\frac{A(f)\cdot B(f)\cdot C_2(f)\cdot D_2(f)}{\frac{1}{s}\cdot(A(f)\cdot B(f)\cdot C_1(f)\cdot D_1(f))}\cdot Area\right)\\
&= Re\left(s\cdot C_2(f)\cdot Area\cdot\frac{1}{C_1(f)}\cdot\frac{D_2(f)}{D_1(f)}\right)
\end{aligned}
\tag{15}
$$

Since the two measurements can be performed in only minutes, the transfer function of the microphone, $D(f)$, is not likely to change in a significant way, i.e. $D_1(f)$ and $D_2(f)$ are expected to be almost identical, which in turn reduces the ratio of $D_2(f)$ and $D_1(f)$ in equation 15 to unity. Provided, that the imaginary part of $p_1(f)$ is much larger than the real part, then $C_1(f)$ is a constant and a real number under most acoustic situations. If these prerequisites are fulfilled, then equation 15 reduces to equation 9 except for the constant factor of $1/C_1(f)$, which is a real number, provided that the real part of $p_1(f)$ is negligible compared to the imaginary part of $p_1(f)$. It might be stressed that $C_2(f)$ in equation 15 is similar to $C(f)$ *in equation 9*, because the sound pressure can be measured at any point in front of the diaphragm/piston as long as equation 4 is fulfilled.

One example where $C_1(f)$ is not constant, is when the diaphragm/piston is moved from free field (4π) to an infinite baffle (2π). In this situation, the sound pressure is doubled in any position, which doubles $C_1(f)$ effectively. But for most practically found placements of a diaphragm/piston relative to the surfaces of a room, this does not offer a problem when measuring the radiation resistance.

3.2 Using one microphone and approximating the acceleration by a difference between two sound pressures

A way to improve the approximation of the acceleration of the diaphragm/piston is to calculate the difference between the two sound pressures, $p_1(f)$ and $p_2(f)$. The purpose of doing so can be seen in figure 6, where the complex sound pressures, $p_1(f)$ and $p_2(f)$, are shown after they have been divided by the velocity, v. One should take notice in the fact, that the real part of $p_1(f)$ and $p_2(f)$ are equal provided that equation 4 is fulfilled. This is stated in equation 14.

Subtracting $p_2(f)$ from $p_1(f)$ will produce a result, which is a pure imaginary number, i.e. parallel to the imaginary axis and perpendicular to the velocity, v. It

should be stressed that this is true without assuming anything regarding the ratio between the imaginary part and the real part of the sound pressures, $p_1(f)$ and $p_2(f)$.

One might then use $p_1(f)-p_2(f)$ to approximate the velocity of the diaphragm/piston, by integrating this sound pressure difference (dividing by $s = j2\pi f$, i.e. the Laplace operator). Section 3.1 gives further details regarding the involved transfer functions, $A(f)$, $B(f)$ etc., and integration of sound pressure. Implementing this in equation 15 leads to equation 16, where the approximation of the radiation resistance is named, $R_{r,b}(f)$.

$$
\begin{aligned}
R_{r,b}(f) &= Re\left(\frac{p_{2,m}(f)}{\frac{1}{s}\cdot(p_{1,m}(f)-p_{2,m}(f))}\cdot Area\right)\\
&= Re\left(\frac{A(f)\cdot B(f)\cdot C_2(f)\cdot D_2(f)\cdot Area}{\frac{1}{s}\cdot(A(f)\cdot B(f)\cdot C_1(f)\cdot D_1(f)-A(f)\cdot B(f)\cdot C_2(f)\cdot D_2(f))}\right)\\
&= Re\left(s\cdot C_2(f)\cdot Area\cdot\frac{1}{\frac{D_1(f)}{D_2(f)}\cdot C_1(f)-C_2(f)}\right)
\end{aligned}
\tag{16}
$$

The ratio of $D_1(f)$ and $D_2(f)$ is expected to reduce to unity, provided that the two measurements are performed within minutes. In this case, the difference in the denominator reduces to the difference between $C_1(f)$ and $C_2(f)$, which is a constant real number as long as position #1 and position #2 obeys equation 4. This constant real number is a function of the chosen positions: #1 and #2, which means that these positions should be fixed, because changing position #1 and/or position #2 will change this constant.

An additional advantage of equation 16 compared to equation 15, is the freedom to choose the numerator: $p_1(f)$, $p_2(f)$ or even $(p_1(f) + p_2(f))$. This has no meaning in equation 15, where the approximation of the radiation resistance reduces to zero, if same sound pressure is used both in the numerator and in the denominator.

The advantage of using $p_1(f)$ is that the reflections are measured with a reduced error if the sound pressure is measured closer to the surface of the diaphragm/piston, see equation 13. Regarding the denominator, i.e. the difference between the two sound pressures, then the reflections have a tendency to contribute equally to the two sound pressures, which leaves the difference almost unaffected by the reflections. Equation 17 gives this version, where the approximation of the radiation resistance is named, $R_{r,c}(f)$.

$$
\begin{aligned}
R_{r,c}(f) &= Re\left(\frac{p_{1,m}(f)}{\frac{1}{s}\cdot(p_{1,m}(f)-p_{2,m}(f))}\cdot Area\right)\\
&= Re\left(\frac{A(f)\cdot B(f)\cdot C_1(f)\cdot D_1(f)\cdot Area}{\frac{1}{s}\cdot(A(f)\cdot B(f)\cdot C_1(f)\cdot D_1(f)-A(f)\cdot B(f)\cdot C_2(f)\cdot D_2(f))}\right) \qquad (17)\\
&= Re\left(s\cdot C_1(f)\cdot Area\cdot\frac{1}{C_1(f)-\frac{D_2(f)}{D_1(f)}\cdot C_2(f)}\right)
\end{aligned}
$$

One might also use both sound pressures in the numerator, which should reduce the noise components in the measured sound pressure to be used in the numerator, see equation 18. From this it can be seen, that this approximation, $R_{r,d}(f)$, is simply the mean of the two recent approximations, i.e. $R_{r,b}(f)$ and $R_{r,c}(f)$.

$$
\begin{aligned}
R_{r,d}(f) &= Re\left(\frac{\frac{1}{2}(p_{1,m}(f)+p_{2,m}(f))}{\frac{1}{s}\cdot(p_{1,m}(f)-p_{2,m}(f))}\cdot Area\right)\\
&= Re\left(\frac{\frac{1}{2}(A(f)\cdot B(f)\cdot C_1(f)\cdot D_1(f)+A(f)\cdot B(f)\cdot C_2(f)\cdot D_2(f))}{\frac{1}{s}\cdot(A(f)\cdot B(f)\cdot C_1(f)\cdot D_1(f)-A(f)\cdot B(f)\cdot C_2(f)\cdot D_2(f))}\cdot Area\right) \qquad (18)\\
&= Re\left(\frac{1}{2}\cdot s\cdot Area\cdot\frac{C_1(f)+\frac{D_2(f)}{D_1(f)}\cdot C_2(f)}{C_1(f)-\frac{D_2(f)}{D_1(f)}\cdot C_2(f)}\right)\\
&= \frac{1}{2}(R_{r,b}(f)+R_{r,c}(f))
\end{aligned}
$$

3.3 Using two microphones and approximating the acceleration by a sound pressure

If two microphones are used, then both sound pressures, $p_1(f)$ and $p_2(f)$, can be measured in one measurement. But now two different transfer functions, *MicA(f)* and *MicB(f)*, are involved in the measurement, i.e. one transfer function for each microphone: microphone A and microphone B. Therefore the measurements have to be performed two times: the second time with the two microphones physically interchanged (they swap physical position). This procedure is illustrated in figure 7.

In this figure, microphone A is placed in position #1 (close to the surface of the diaphragm/piston) and microphone B is placed in position #2 (further away from the surface of the diaphragm/piston) during the first measurement. After this

measurement, a physical interchange of the two microphones is performed, i.e. microphone A is now in position #2 and microphone B is in position #1 and the measurement is repeated.

Figure 8 shows a diagram of the signals and transfer functions involved in the measurement procedure. Unit *A* is a power amplifier. Unit *B* is the electrical and mechanical parts of the loudspeaker driver unit and the acoustic influence of the cabinet enclosure. Unit C_1 is the transfer function from membrane acceleration to sound pressure in position #1. Unit C_2 is the transfer function from membrane acceleration to sound pressure in position #2. $MicA_1$ is the transfer function of microphone A during the first measurement, $MicA_2$ is the transfer function of microphone A during the second measurement. Similar names are given to the transfer functions of microphone B: $MicB_1$ and $MicB_2$.

Signal 1 is the input voltage at line level, and signal 2 is the voltage on the terminals of the loudspeaker driver unit. Signal 3 is the acceleration of the diaphragm/piston, $p_1(f)$ is the sound pressure at position #1, $p_2(f)$ is the sound pressure at position #2. $p_{1,m1}(f)$ and $p_{2,m1}(f)$ are the measured sound pressures from the first measurement, and $p_{1,m2}(f)$ and $p_{2,m2}(f)$ are the measured sound pressures from the second measurement.

During both the first and second measurements, the transfer function between the measured sound pressure close to the diaphragm/piston, $p_{1,m}(f)$, and the measured sound pressure further away from the diaphragm/piston, $p_{2,m}(f)$ are calculated. This is given in equation 19. $H_1(f)$ is calculated from the first measurement, and $H_2(f)$ is calculated from the second measurement.

$$H_1(f) = \frac{p_{2,m1}(f)}{p_{1,m1}(f)} = \frac{p_2(f) \cdot MicB_1(f)}{p_1(f) \cdot MicA_1(f)}$$
$$H_2(f) = \frac{p_{2,m2}(f)}{p_{1,m2}(f)} = \frac{p_2(f) \cdot MicA_2(f)}{p_1(f) \cdot MicB_2(f)} \quad (19)$$

If $H_1(f)$ and $H_2(f)$ is multiplied, then the influence of $MicA(f)$ and $MicB(f)$ is removed provided that they don't change from the first measurement to the second measurement, i.e. $MicA_1(f) = MicA_2(f)$ and $MicB_1(f) = MicB_2(f)$. This enables a calculation of $H(f)$, which ideally is independent of the transfer functions of the microphones. See equation 20.

$$H(f) = \sqrt{H_1(f) \cdot H_2(f)} = \frac{p_2(f)}{p_1(f)} \cdot \sqrt{\frac{MicB_1(f)}{MicB_2(f)} \cdot \frac{MicA_2(f)}{MicA_1(f)}} \quad (20)$$

Equation 20 gives an alternative way of obtaining $p_1(f)$ and $p_2(f)$, namely through their ratio, which is sufficient, see equation 15, 16, 17 and 18. It should be stressed, that the advantage of equation 19 and 20 (using two microphones) is that no

constraints is made on the power amplifier or loudspeaker (sound source), e.g. regarding linearity. If the power amplifier and/or the loudspeaker is nonlinear, then the measurements of the transfer functions involved in section 3.1 and 3.2 might be difficult, e.g. if broad band signals are used. This is not the case when using two microphones, because the needed transfer functions, $H_1(f)$ and $H_2(f)$, don't involve the power amplifier or the loudspeaker. Considering figure 8 leads to an important relation between the sound pressures, $p_1(f)$ and $p_2(f)$, and the transfer functions, $C_1(f)$ and $C_2(f)$, which is given in equation 21.

$$\frac{p_2(f)}{p_1(f)} = \frac{C_2(f)}{C_1(f)} \tag{21}$$

If equation 20 is used to obtain the ratio between $p_1(f)$ and $p_2(f)$, then equation 15 is changed into equation 22, where the involved transfer functions relates to figure 8. This approximation to the radiation resistance is named, $R_{r,e}(f)$. Further details about the basis of equation 22 can be found in section 3.1.

$$\begin{aligned} R_{r,e}(f) &= Re(s \cdot H(f) \cdot Area) \\ &= Re\left(s \cdot C_2(f) \cdot Area \cdot \frac{1}{C_1(f)} \cdot \sqrt{\frac{MicB_1(f)}{MicB_2(f)} \cdot \frac{MicA_2(f)}{MicA_1(f)}}\right) \end{aligned} \tag{22}$$

3.4 Using two microphones and approximating the acceleration by a difference between two sound pressures

Using two microphones can also improve the procedure given in section 3.2, i.e. removing the power amplifier and loudspeaker from the measurement chain. Using equation 20 to obtain the ratio between $p_1(f)$ and $p_2(f)$, changes equation 16 to equation 23. Now the approximation of the radiation resistance is named $R_{r,f}$

$$\begin{aligned} R_{r,f}(f) &= Re\left(s \cdot \frac{H(f)}{1 - H(f)} \cdot Area\right) \\ &= Re\left(s \cdot C_2(f) \cdot Area \cdot \frac{1}{\sqrt{\frac{MicB_2(f)}{MicB_1(f)} \cdot \frac{MicA_1(f)}{MicA_2(f)}} \cdot C_1(f) - C_2(f)}\right) \end{aligned} \tag{23}$$

Similar to the above change, equation 17 is changed into equation 24, and equation 18 is changed into equation 25. These two approximations are named $R_{r,g}(f)$ and $R_{r,h}(f)$.

$$R_{r,g}(f) = Re\left(s \cdot \frac{1}{1 - H(f)} \cdot Area\right)$$
$$= Re\left(s \cdot C_1(f) \cdot Area \cdot \frac{1}{C_1(f) - \sqrt{\frac{MicB_1(f)}{MicB_2(f)} \cdot \frac{MicA_2(f)}{MicA_1(f)}} \cdot C_2(f)}\right) \tag{24}$$

$$R_{r,h}(f) = Re\left(\frac{1}{2} \cdot s \cdot \frac{1 + H(f)}{1 - H(f)} \cdot Area\right)$$
$$= Re\left(\frac{1}{2} \cdot s \cdot Area \cdot \frac{C_1(f) + \sqrt{\frac{MicB_1(f)}{MicB_2(f)} \cdot \frac{MicA_2(f)}{MicA_1(f)}} \cdot C_2(f)}{C_1(f) - \sqrt{\frac{MicB_1(f)}{MicB_2(f)} \cdot \frac{MicA_2(f)}{MicA_1(f)}} \cdot C_2(f))}\right) \tag{25}$$
$$= \frac{1}{2}(R_{r,f}(f) + R_{r,g}(f))$$

3.5 Separate calibration using two microphones

All the procedures described in sections 3.1 to 3.4 are characterized by an implicit calibration of the microphone or microphones. Explicit calibration or separate calibration, involves two types of measurements, calibration measurement(s) and the actual measurement(s) of the radiation resistance. The order of these types of measurements is not important.

Separate calibration applies only to two or more microphones. If two microphones are used, then the calibration might be performed by enabling the two microphones to measure the sound pressure at the same or almost the same position. This can be achieved by calibration of two closely separated microphones, or by interchanging two microphones at the same position.

For example, microphone A can be fixed at position #1 and microphone B can be moved from position #2 to position #1 (just next to microphone A) and back to position #2. In this case, the calibration takes place by moving microphone B to position #1, which places the two microphones just next to each other. Assuming that the sound pressure is identical at the two microphones enables a relative calibration of the two microphones. Equation 26 calculates a calibration factor, *cal(f)*.

$$cal(f) = \frac{MicA_c(f)}{MicB_c(f)} \tag{26}$$

$MicA_c(f)$ and $MicB_c(f)$ are the transfer functions of the microphones at the calibration time. After calibration, microphone B returns to position #2 and the sound pressures at position #1 and #2 are measured: $p_{2,m}(f)$ and $p_{1,m}(f)$, i.e. microphone A measures $p_{1,m}(f)$ and microphone B measures $p_{2,m}(f)$. The transfer function, $H_c(f)$, which ideally is independent of the transfer functions of the microphones can then be calculated from equation 27.

$$H_C(f) = \frac{p_{2,m}(f)}{p_{1,m}(f)} \cdot cal(f) = \frac{p_2(f)}{p_1(f)} \cdot \frac{MicBm(f)}{MicAm(f)} \cdot \frac{MicAc(f)}{MicBc(f)} \tag{27}$$

$MicA_m(f)$ and $MicB_m(f)$ are the transfer functions of the microphones at the measurement time. $H_c(f)$ can now be used in the same way as $H(f)$ in equation 20 (see section 3.3 and 3.4).

More than two microphones might be used even when only two positions are considered. For example a third microphone might be moved from position #1 to #2, while microphone A and B are fixed in position #1 and #2. The relative calibration of microphone A and B might then be achieved by performing measurements with the third microphone placed first next to microphone A and then next to microphone B.

3.6 Measuring the sound pressure at three or more positions

In section 3.1 to 3.5, the sound pressures are measured at two positions using one, two or more microphones. But the fundamental idea of this invention: "Obtaining the radiation resistance of a piston by measuring sound pressures only" is not limited to two positions. Using three or more positions is a simple extension of using two positions.

For example, if three positions are used, then the acceleration/velocity (denominator of equation 1) might be approximated by subtracting the sound pressures in position #1 and #2, while the sound pressure (numerator of equation 1) is measured in position #3. One approach would be to choose position #3 as half way between positions #1 and #2.

In order to improve the measurement of the contributions of reflections, even more positions near the surface of the diaphragm/piston might be used, and the sound pressures measured here should then be added/averaged. Both numerator and denominator of equation 1 can take advantage of using three or more positions, but using more positions does not change the fundamental principle of the invention: "measuring only one physical parameter, sound pressure, in order to obtain the radiation resistance".

4 Conclusion

The proposed Moving Microphone System (MMS) is a method to obtain the radiation resistance seen from a piston, e.g. a loudspeaker diaphragm (active radiator or passive radiator) or a loudspeaker port, which can be modeled as a fictive piston. MMS employs one, two or more microphones to measure the sound pressure in two or more positions near the surface of the piston. The invention is exemplified by two positions in detail (sections 3.1 to 3.4), while three and more positions is descibed in a separate section (section 3.6).

If two positions are to be used, then one should choose the two positions in such a way that the amplitude of the sound pressures differ significantly, e.g. of the order of 2 dB - 6 dB. This can be obtained in numerous ways. One example can be given: the two positions could be placed on the rotational symmetry axis of a loudspeaker diaphragm, i.e. perpendicular to the surface. The distances from the surface of the diaphragm along the axis could be chosen to be 2 cm and 6 cm, see figure 5.

An essential characteristic of a MMS is the physical movement of the microphone or microphones. When one microphone is used, then this microphone is physically moved (translation or rotation) from position #1 to position #2 to enable measurement of both sound pressures using a single microphone, see figure 5. When two microphones are used, then the two microphones must swap position between the two measurements, which enables both sound pressures to be measured by both microphones, see figure 7. As an alternative means to move/swap the microphone(s), one can use pipes from a centrally placed microphone(s) together with some switch/valve, which controls where the sound pressure(s) is/are measured.

Swaping the positions of two microphones enables an implicit calibration, because the measurements are repeated after swaping the positions. Alternatively explicit/separate calibration might be performed, which opens for the possibility of fixing the position of one of the microphone and only having to move the other. Section 3.5 descibes this possibility.

8 different ways to calculate the radiation resistance have been given, depending on the number of microphones used and chosen way to approximate the acceleration/velocity of the diaphragm. These are summarised below.

One microphone:

$$R_{r,a}(f) = Re\left(s \cdot \frac{p_{2,m}(f)}{p_{1,m}(f)} \cdot Area\right)$$

$$R_{r,b}(f) = Re\left(s \cdot \frac{p_{2,m}(f)}{p_{1,m}(f) - p_{2,m}(f)} \cdot Area\right)$$

$$R_{r,c}(f) = Re\left(s \cdot \frac{p_{1,m}(f)}{p_{1,m}(f) - p_{2,m}(f)} \cdot Area\right)$$

$$R_{r,d}(f) = Re\left(\frac{1}{2} \cdot s \cdot \frac{p_{1,m}(f) + p_{2,m}(f)}{p_{1,m}(f) - p_{2,m}(f)} \cdot Area\right)$$

Two microphones:

$$R_{r,e}(f) = Re\left(s \cdot \sqrt{\frac{p_{2,m1}(f)}{p_{1,m1}(f)} \cdot \frac{p_{2,m2}(f)}{p_{1,m2}(f)}} \cdot Area\right)$$

$$R_{r,f}(f) = Re\left(s \cdot \frac{1}{\sqrt{\frac{p_{1,m1}(f)}{p_{2,m1}(f)} \cdot \frac{p_{1,m2}(f)}{p_{2,m2}(f)}} - 1} \cdot Area\right)$$

$$R_{r,g}(f) = Re\left(s \cdot \frac{1}{1 - \sqrt{\frac{p_{2,m1}(f)}{p_{1,m1}(f)} \cdot \frac{p_{2,m2}(f)}{p_{1,m2}(f)}}} \cdot Area\right)$$

$$R_{r,h}(f) = Re\left(\frac{1}{2} \cdot s \cdot \frac{1 + \sqrt{\frac{p_{2,m1}(f)}{p_{1,m1}(f)} \cdot \frac{p_{2,m2}(f)}{p_{1,m2}(f)}}}{1 - \sqrt{\frac{p_{2,m1}(f)}{p_{1,m1}(f)} \cdot \frac{p_{2,m2}(f)}{p_{1,m2}(f)}}} \cdot Area\right)$$

The advantage of all 8 methods is that if the transducers (microphones) can be assumed to stay constant, i.e. constant transfer function, between the two measurements involved then the calculated radiation resistance will not be under influence from the transfer functions of the microphones. The choice of

microphone now becomes much less important since the total measurement can be performed within a few minutes and most microphones will have a constant transfer function during such a short period.

The fundamental principle of the invention is: "measuring only one physical parameter: sound pressure in order to obtain the radiation resistance". This enables usage of the same type of transducer to measure both the movement of the diaphragm/piston and the sound pressure in front of the diaphragm/piston, which in turn enables explicit or implicit calibration of these transducers.

5 References

[1] Knud Rasmussen, "Sound Fields" (Danish title: "Lydfelter"), Laboratory of Acoustics, Technical University of Denmark, 1990.

6 Figures

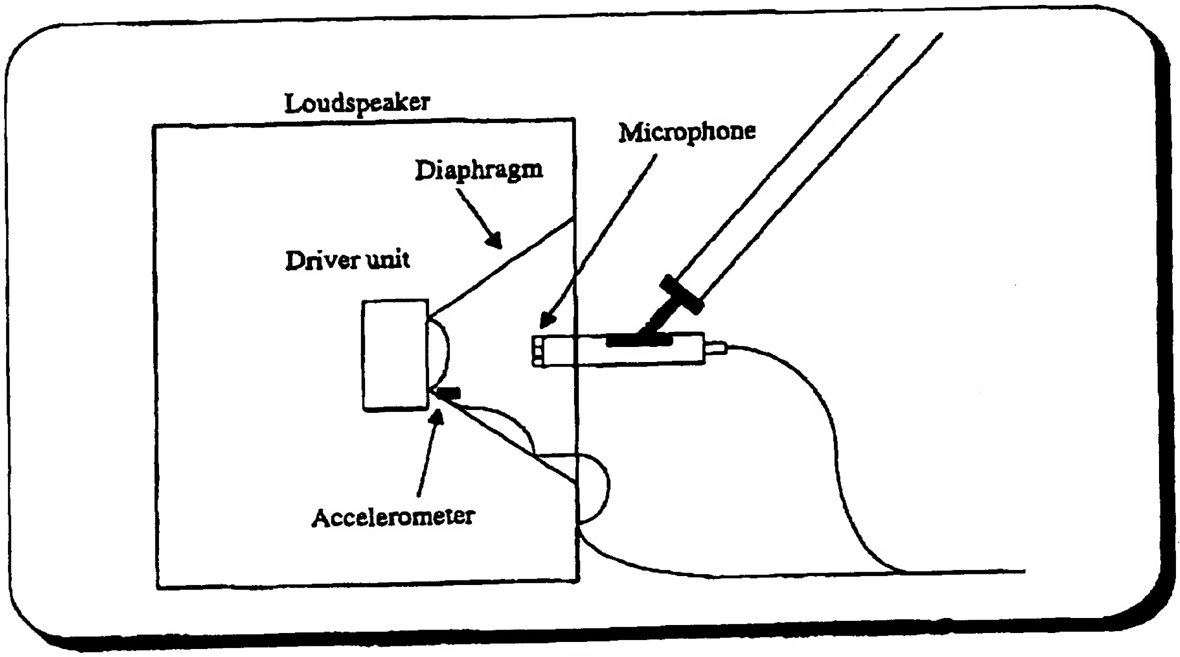

Figure 1

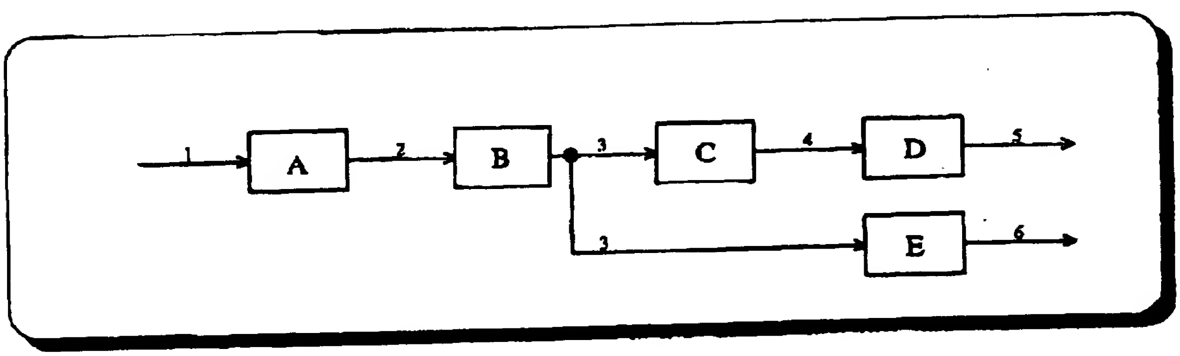

Figure 2

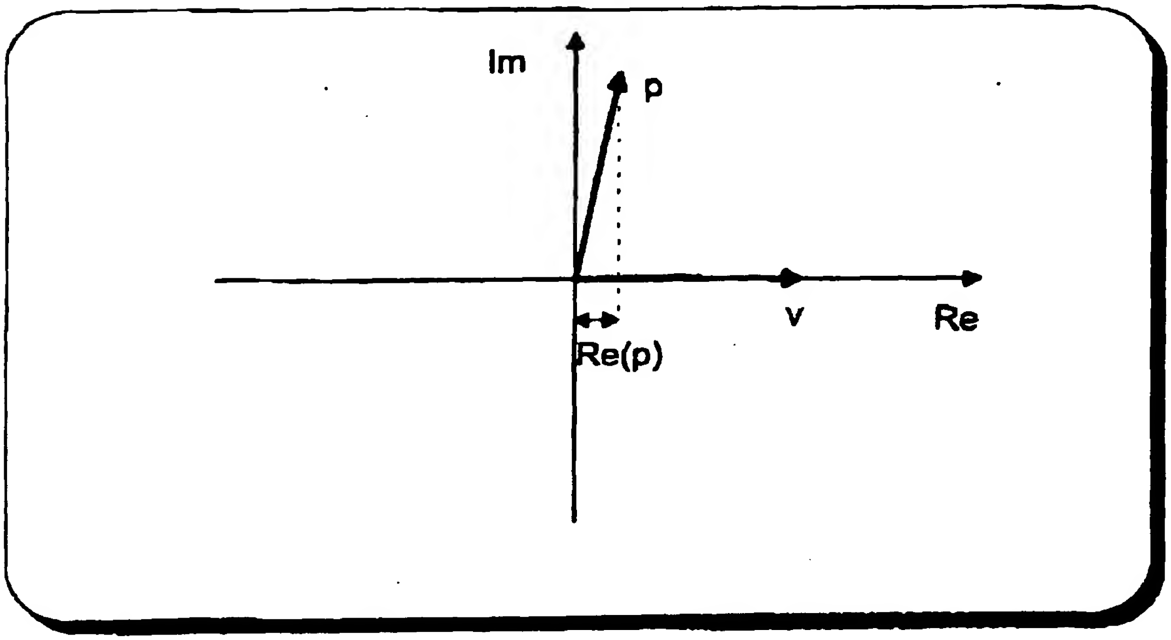

Figure 3

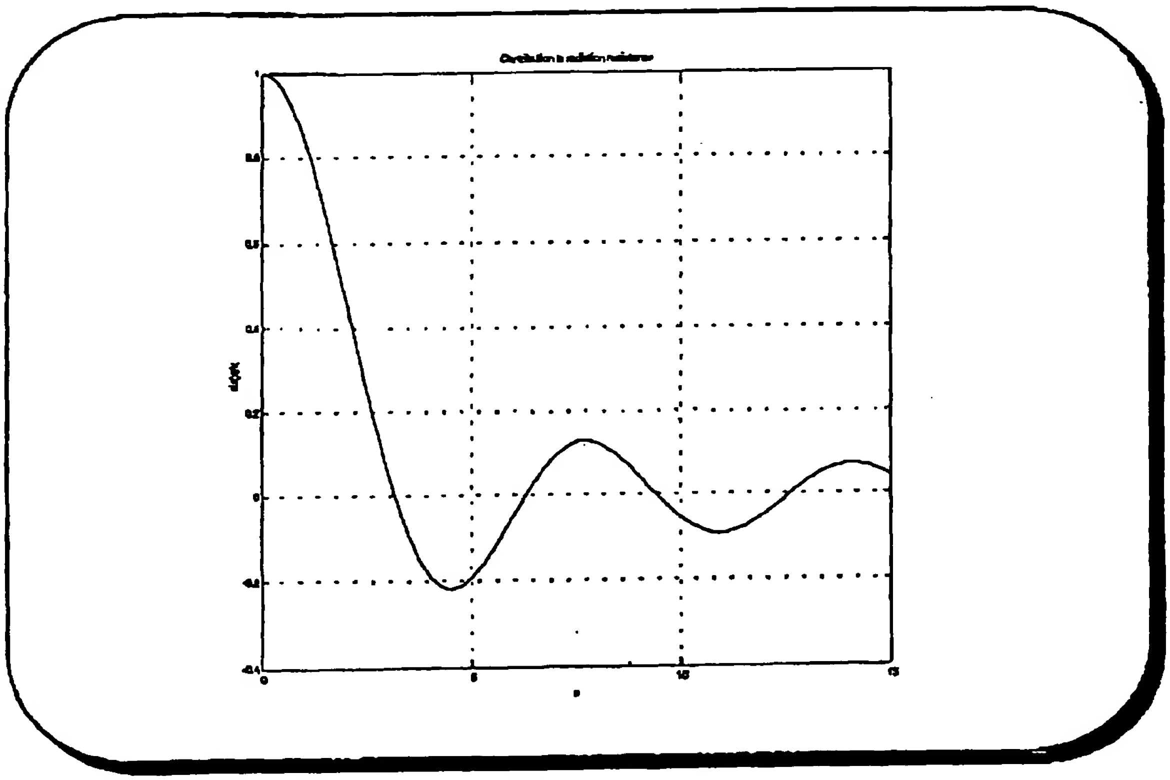

Figure 4

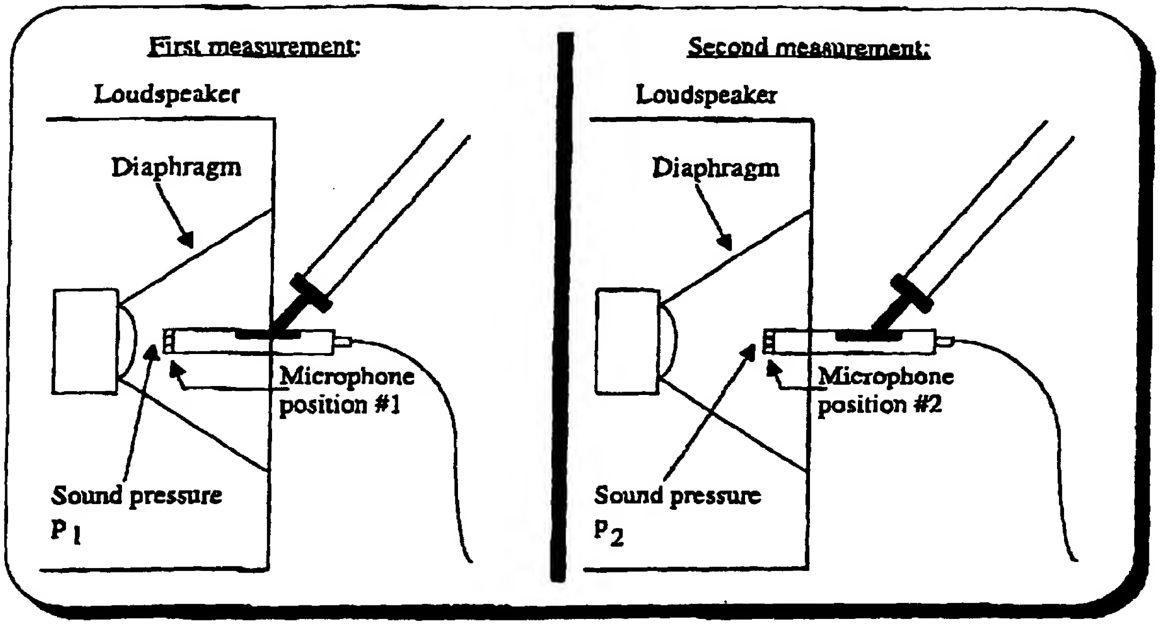

Figure 5

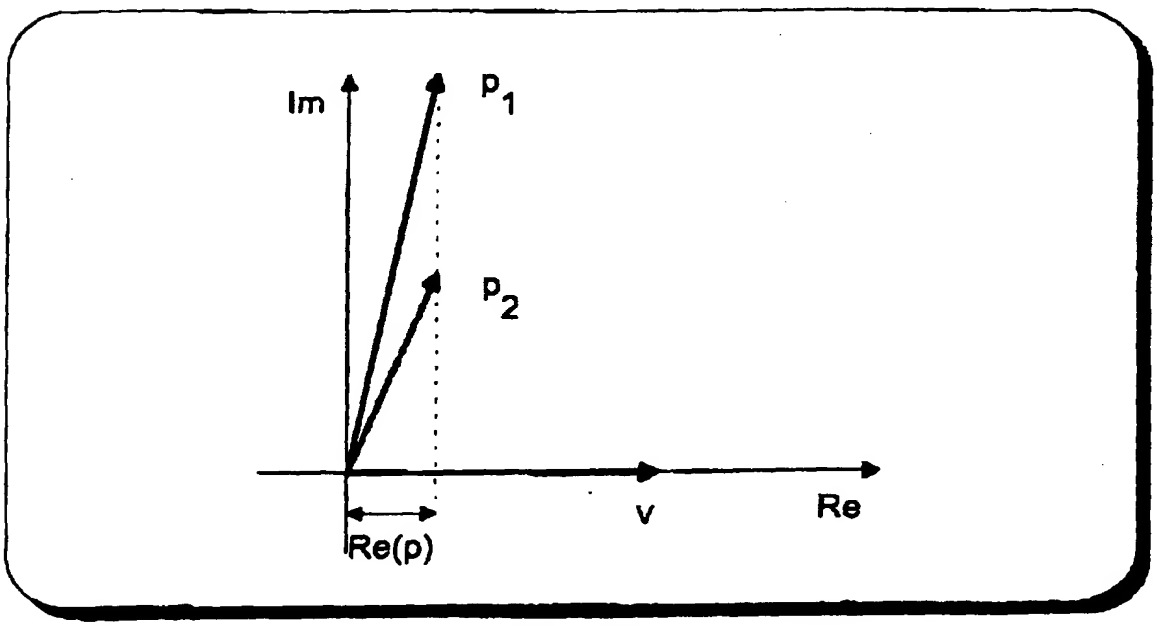

Figure 6

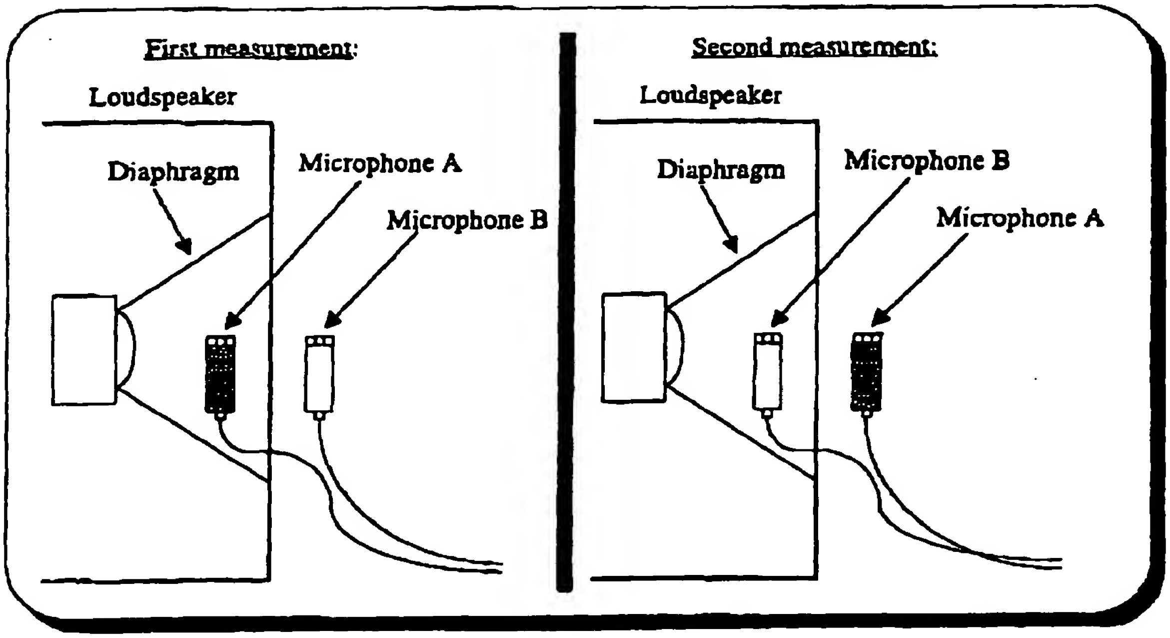

Figure 7

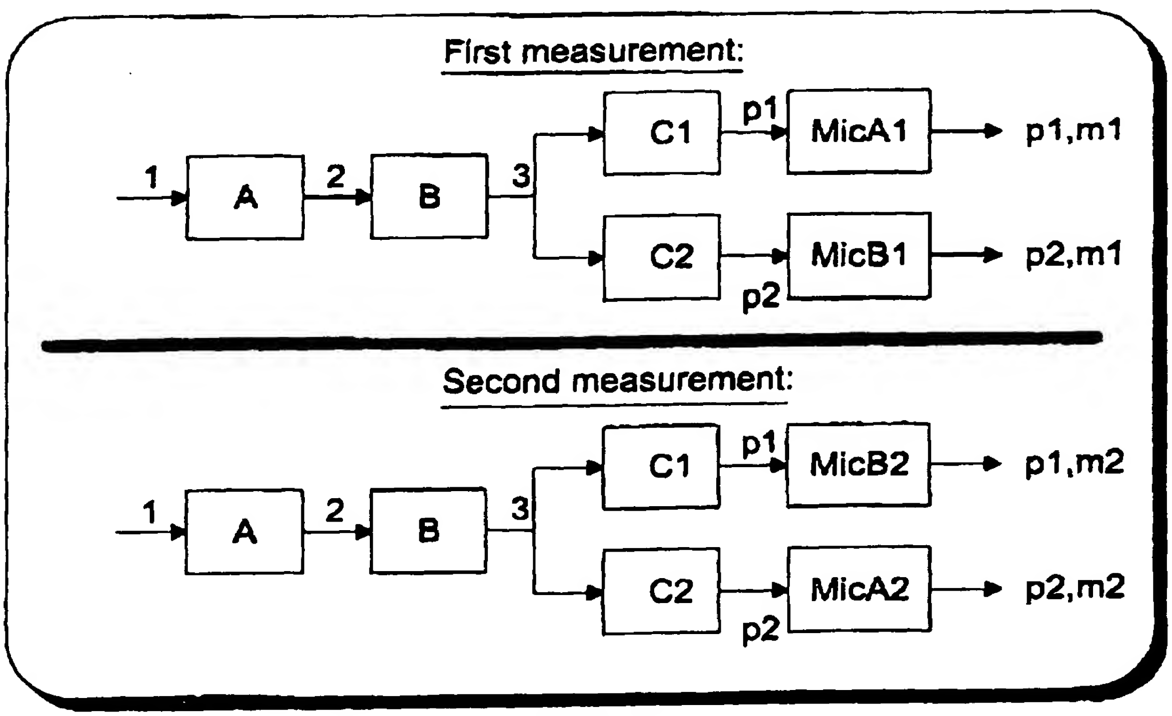

Figure 8